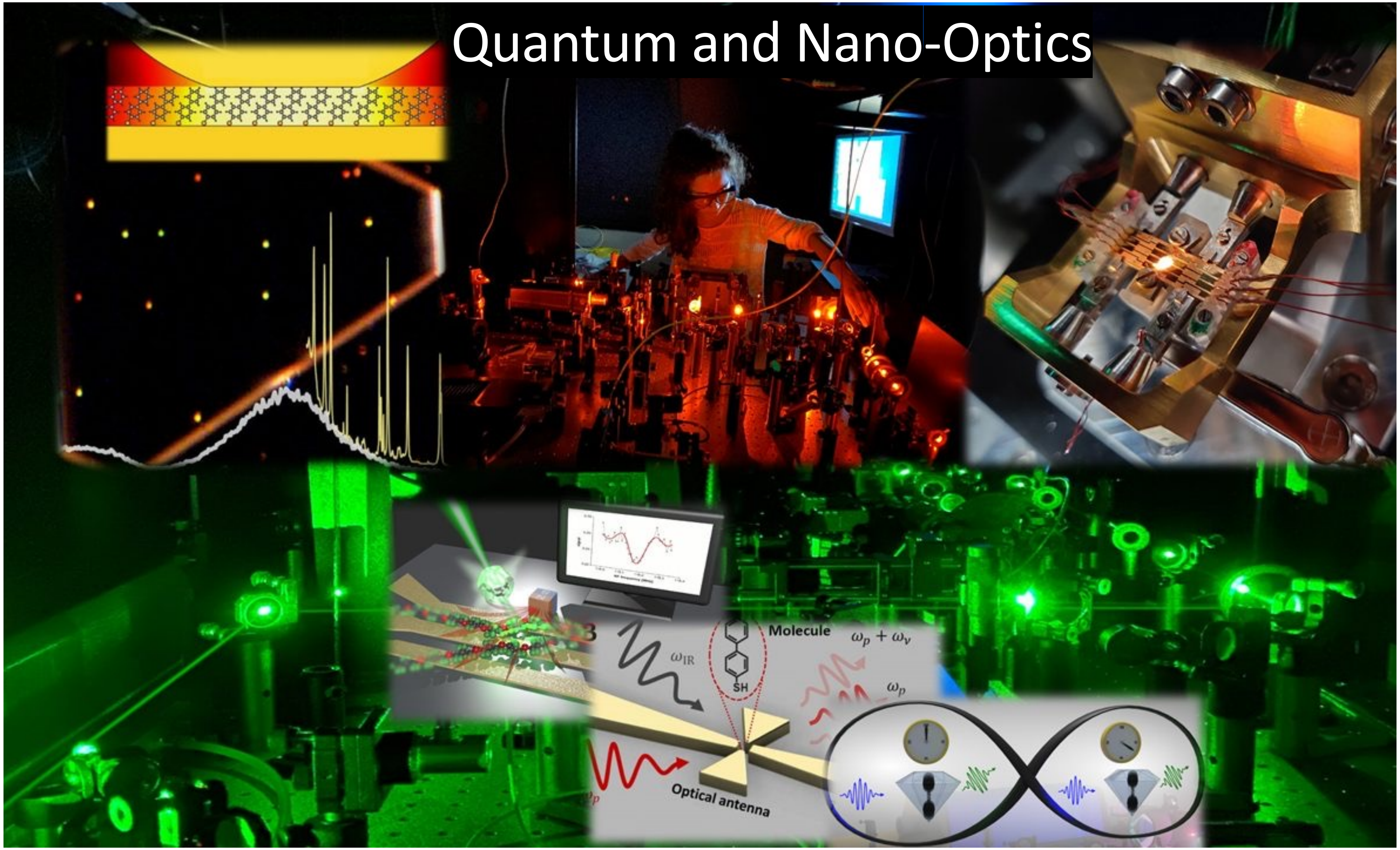
Nonlinear Optics

For Quantum Technologies

Week 01

https://www.epfl.ch/labs/lqno/

- Light-matter interaction inside nanocavities
- Nonlinear optics for quantum state generation
- Diamond photonics and quantum sensing



Course outline

- Classical and semi-classical treatment (6 weeks)

1 Key elements of linear optics

2 The nonlinear susceptibility

3 Wave propagation in nonlinear media

- Quantum treatment and applications (7 weeks)

4 Quantization of the electromagnetic field and nonlinear Hamiltonian

5 Spontaneous Parametric Down Conversion and entangled photon pairs

6 Integrated quantum nonlinear optics and applications in quantum technologies

Organisation

- **Thursdays from 9:15 to 13:00 (4 x 45 min)**

13 weeks (holidays on Apr. 24th and May 29th)

- **9:15 – 11:00 Exercise sessions with Dr. Valentin Goblot and Dr. Konstantin Malchow**

- **11:15 – 13:00 Lecture**

- **Written exam (2h)**
  questions on a research paper (shared in advance)
  questions/problems on what was done during the semester

- **Bonus (up to +0.5): Collected homework during the semester (weeks 6 and 14) and contribution to lecture summaries**

Collaborative lecture summaries

- **For each lecture (starting next week), 3 students are assigned the task of independently compiling in LaTeX a short lecture summary**
- **I collect this 3 documents and compile a single summary that is then available to all students (via Overleaf)**
- **Based on the same principle, we will also establish a collaborative "cheat sheet" for the exam. Each summary should be accompanied by a list of a few key formulas to this end**

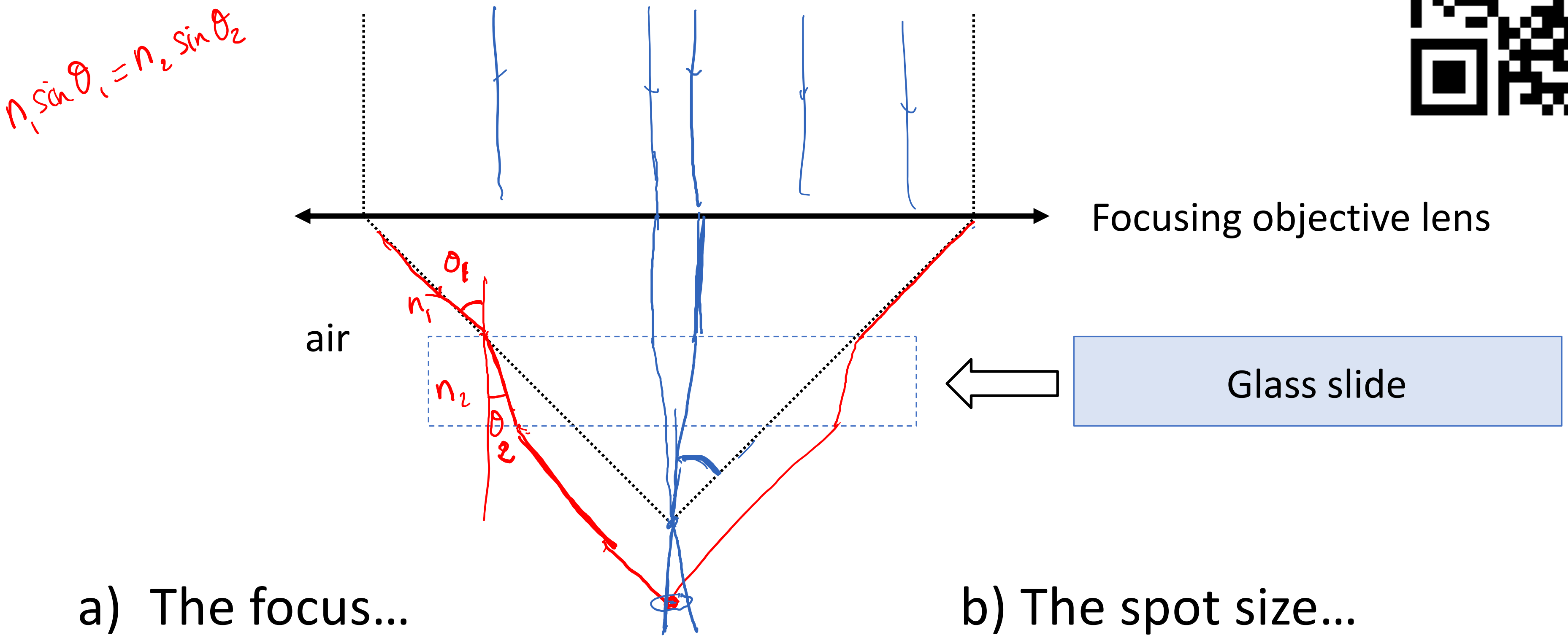
Objectives

- **Describe** the microscopic origin of optical nonlinearities
- **Predict** their macroscopic manifestations
- **Design** a setup based on nonlinear optics for a particular task (frequency conversion, quantum light generation, etc.)
- **Compute** the expected quantum state out of a nonlinear medium
- **Understand** contemporary scientific literature in the field

Warm-up quiz

Checking your background in optics

Question 1



a) The focus…

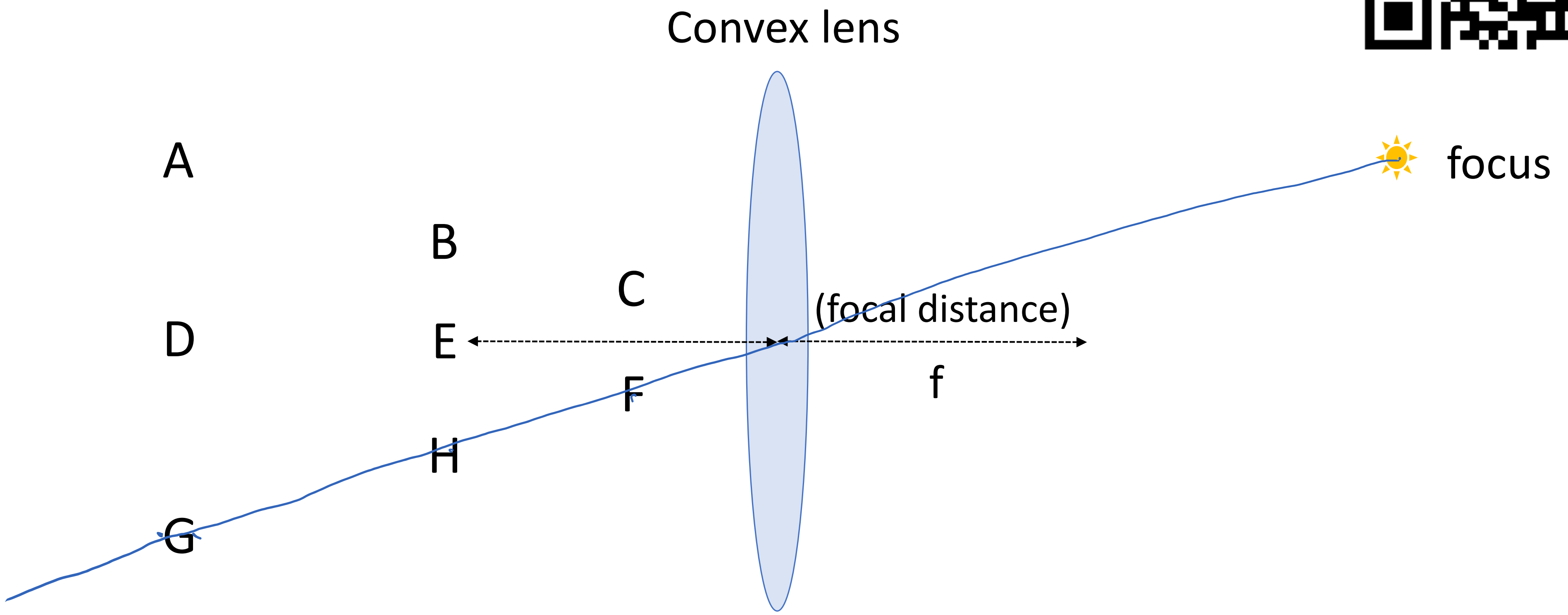
A. Is unchanged
B. Moves upward
C. Moves downward
D. None of the above

b) The spot size…

A. Is unchanged
B. Is increased
C. Is decreased
D. None of the above

Question 2

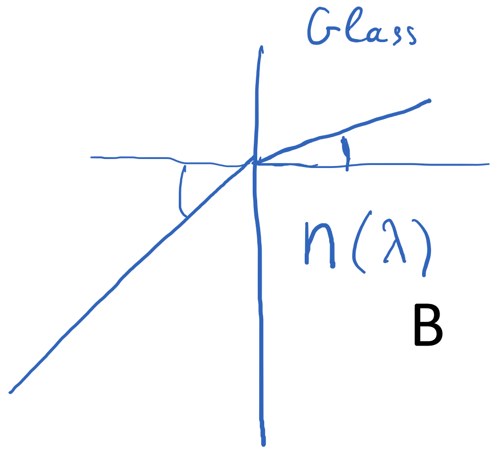
Where is the point light source?

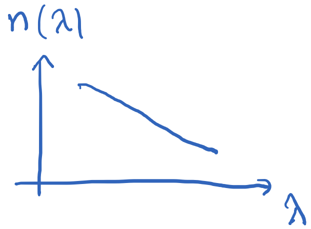


Thin lens equation : $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ ← ok $d_o = d_i = 2f$

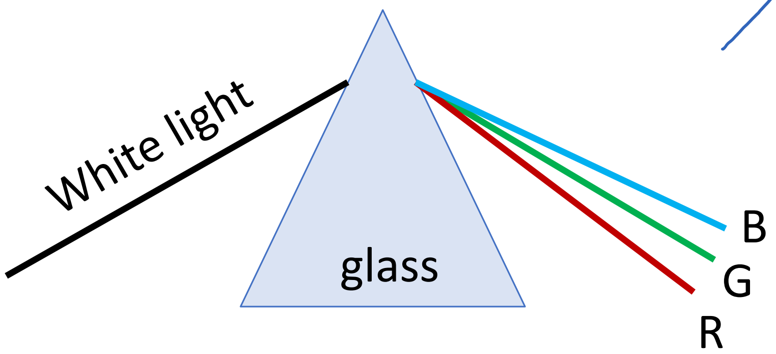
Question 3

What is the correct picture?

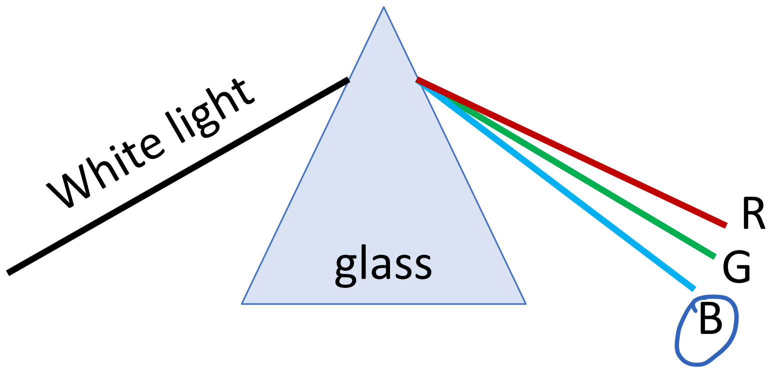




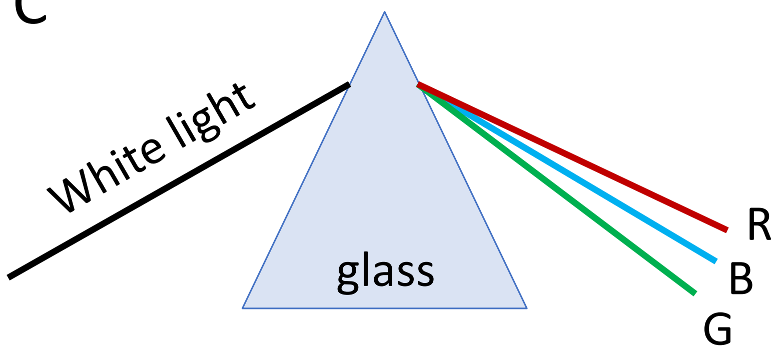
A



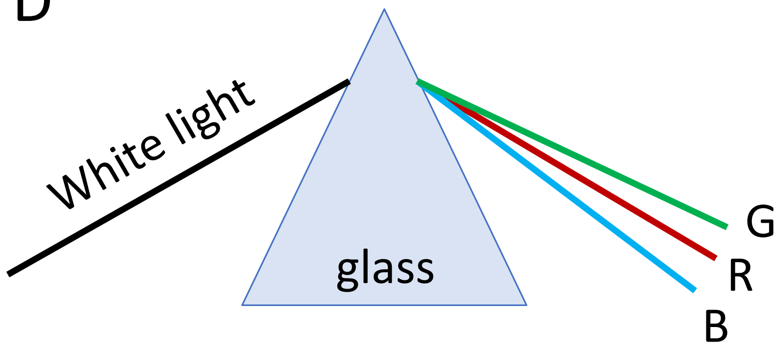
B



C

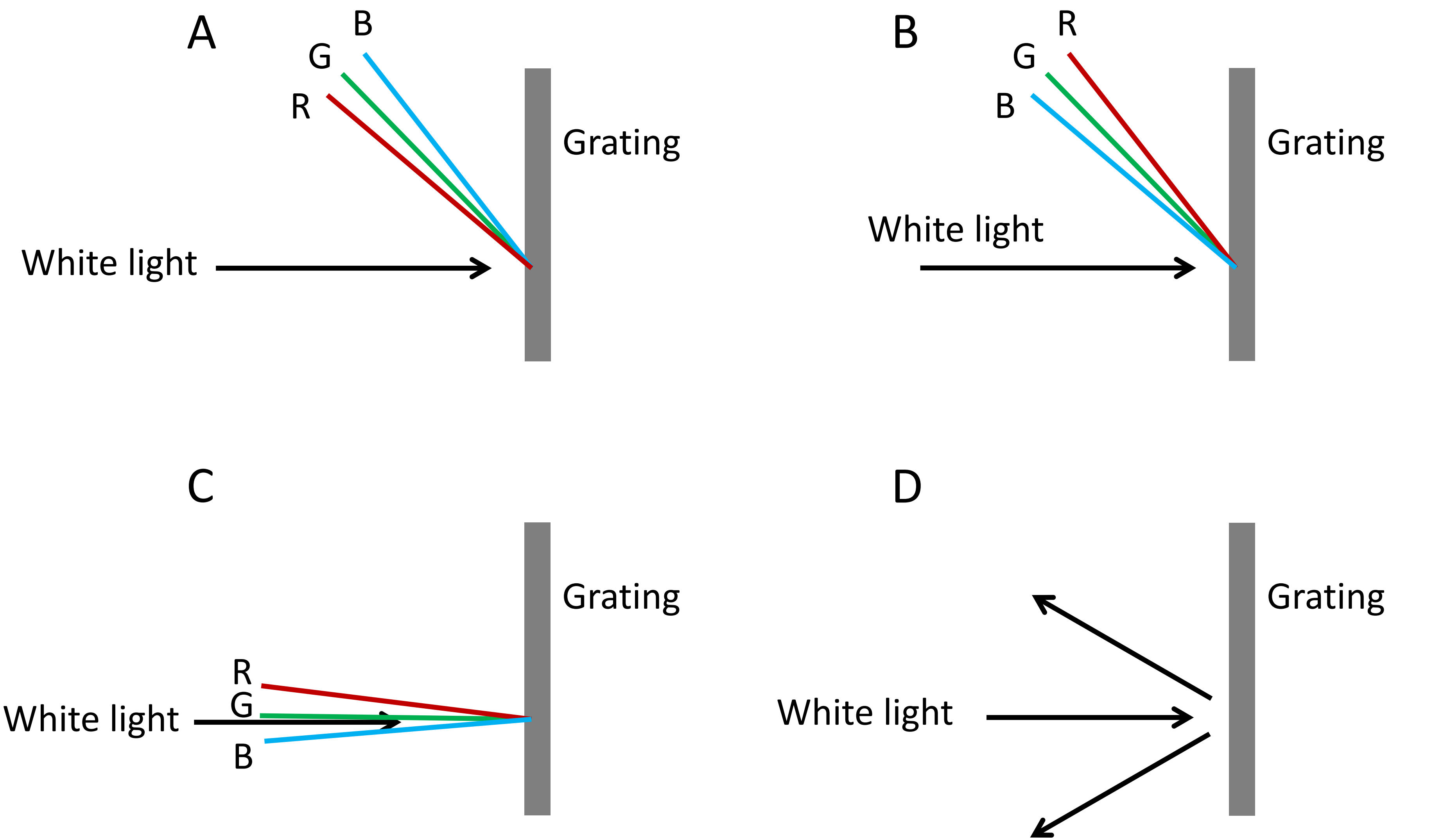


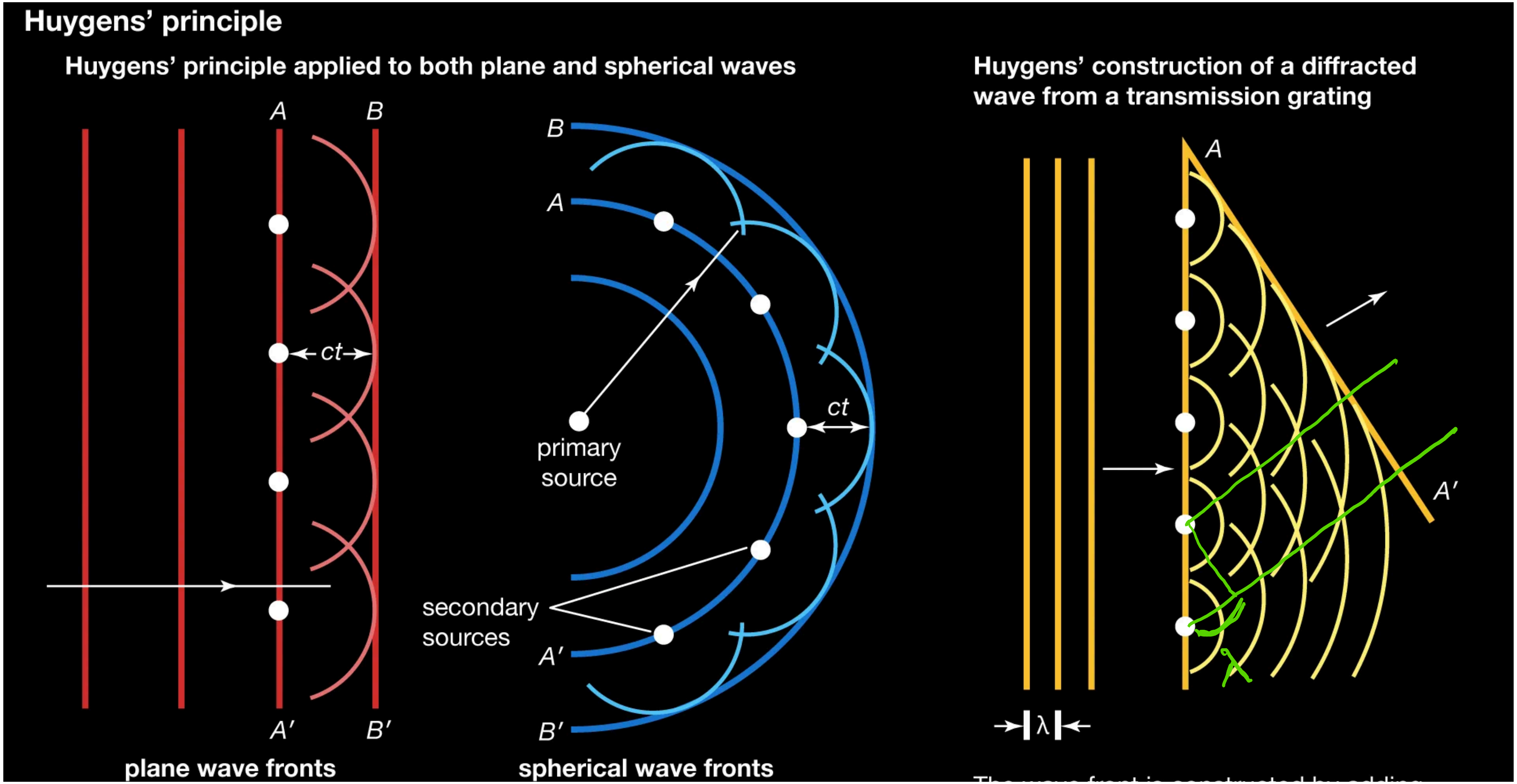
D



Question 4

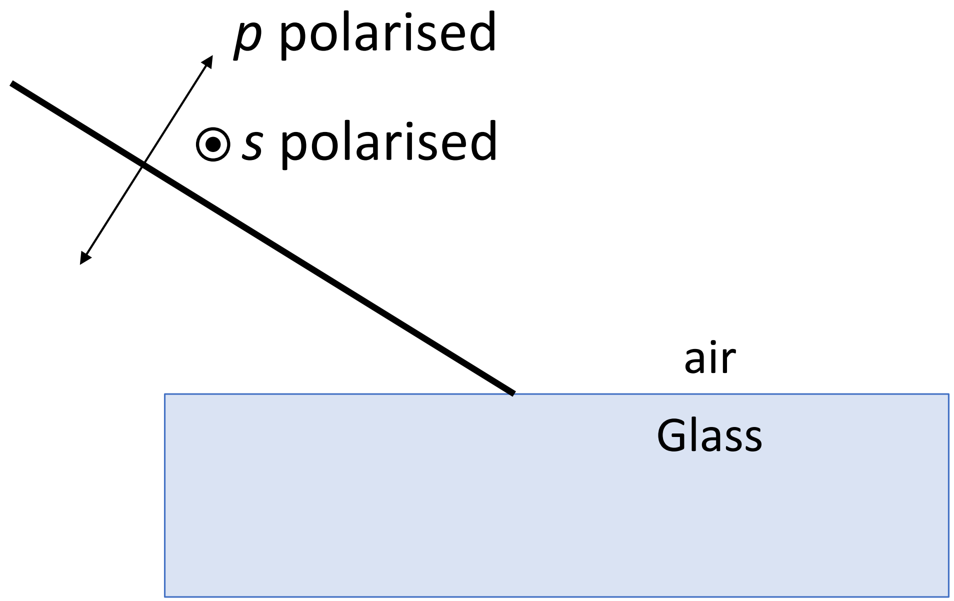
What is the correct picture?





https://www.britannica.com/science/diffraction

Question 5



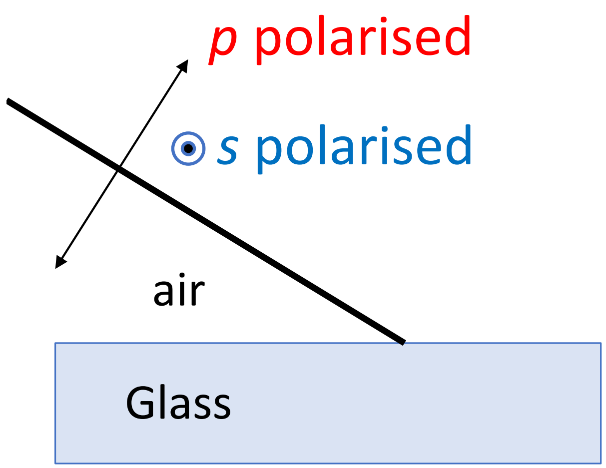
Which is true?

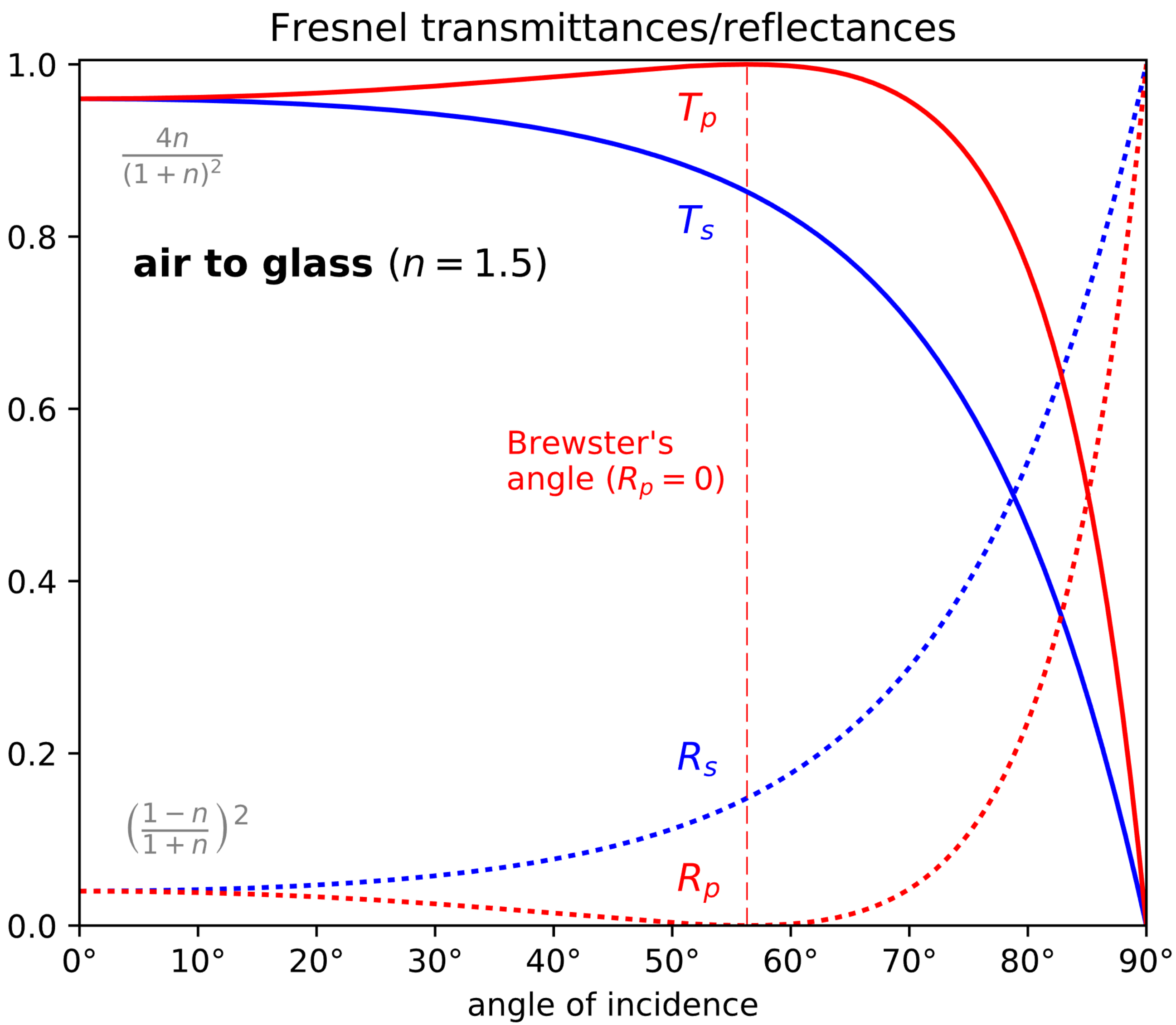
A. *p* is always more transmitted than *s*

B. *s* is always more transmitted than *p*

C. Both polarisations are equally transmitted

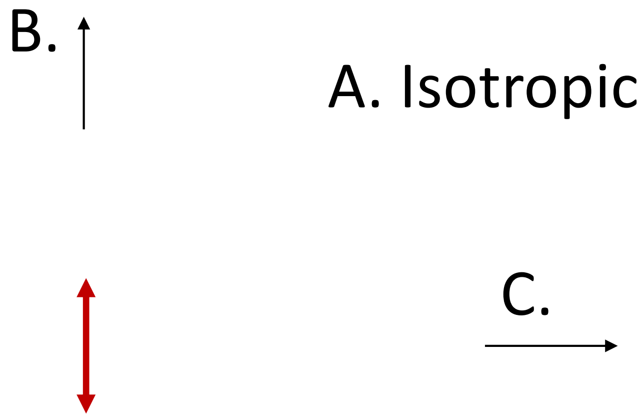
D. The true proposition depends on the incidence angle


p polarised
s polarised
air
Glass


Fresnel transmittances/reflectances
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{4n}{(1+n)^2}$
air to glass ($n = 1.5$)
$\left(\frac{1-n}{1+n}\right)^2$
T_p
T_s
Brewster's angle ($R_p = 0$)
R_s
R_p
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
angle of incidence

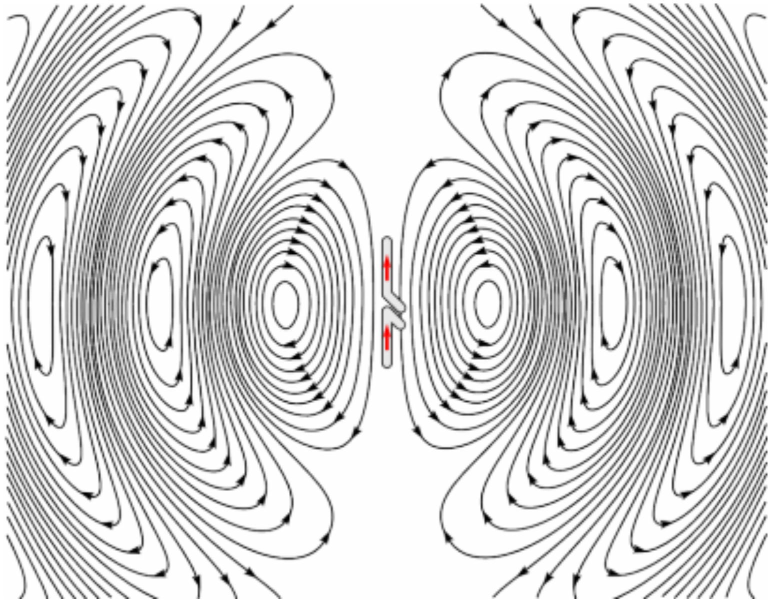
Question 6

In which direction is the electromagnetic radiation stronger?

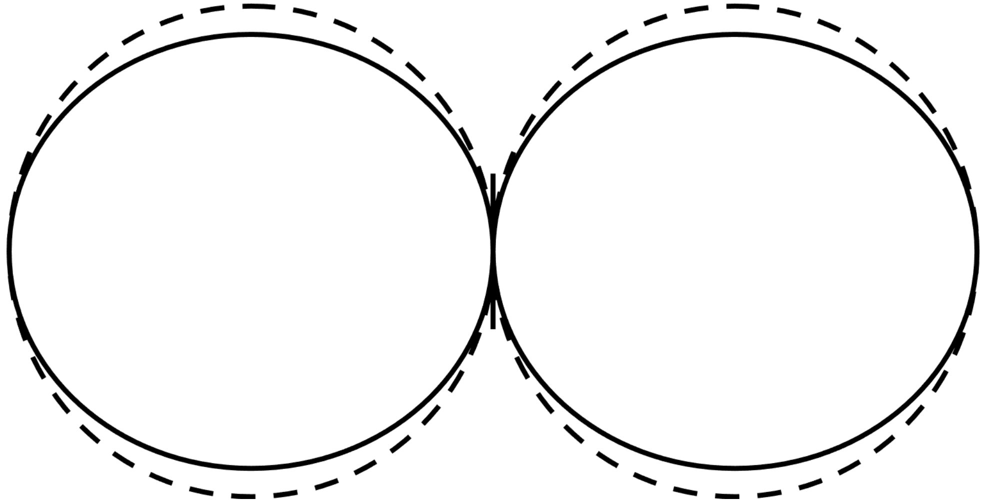


Oscillating electric dipole

D. Another direction



© Wikipedia

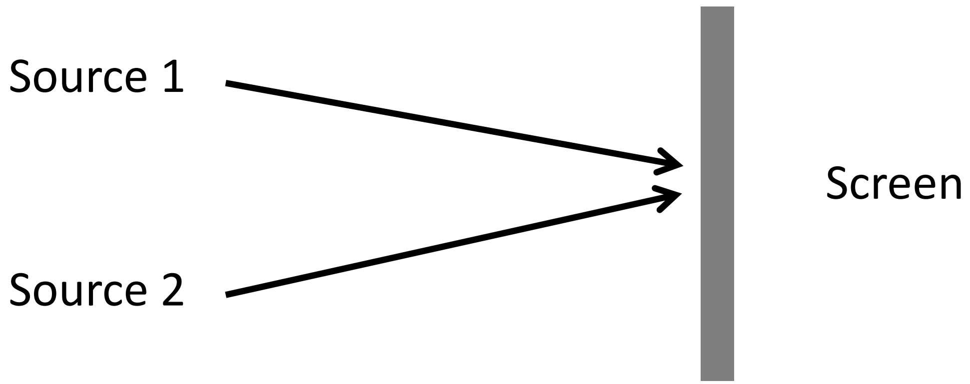


Radiation pattern of the short dipole (dashed line) compared to the half-wave dipole (solid line)

Question 7

In which case(s) do we expect to see interference fringes?
When sources 1 and 2 are two…

A. extended incandescent lights
B. point-like incandescent lights
C. Lasers of different wavelengths
D. Lasers of the same wavelength
E. None of the above



Question 8

What is the result of applying the ladder operator $\hat{a}^\dagger$ on a Fock state $|n\rangle$?

A. $|n-1\rangle$

B. $|n+1\rangle$

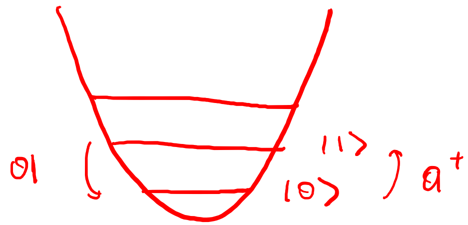
C. $\alpha|n\rangle$, where α is a complex number

D. $\sqrt{n}|n-1\rangle$

E. $\sqrt{n+1}|n+1\rangle$

F. $|0\rangle$

G. None of the above



$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$

$\langle n|a^\dagger a|n\rangle = \| a|n\rangle \|^2$

$= n$

Question 9

What is the state best approximating a strongly attenuated laser pulse?

A. $|1\rangle + \epsilon|2\rangle$

B. $|0\rangle + \epsilon|1\rangle$

C. $|0\rangle\langle 0| + \epsilon|1\rangle\langle 1|$

D. $\epsilon|1\rangle\langle 1|$

E. $|1\rangle\langle 1| + \epsilon|2\rangle\langle 2|$

$|\alpha\rangle \propto |0\rangle + \varepsilon|1\rangle + \frac{\varepsilon^2}{\sqrt{2}}|2\rangle + \ldots$

Question 10

What state best describes unpolarized light?

A. $\frac{1}{2}(|H\rangle + |V\rangle)$

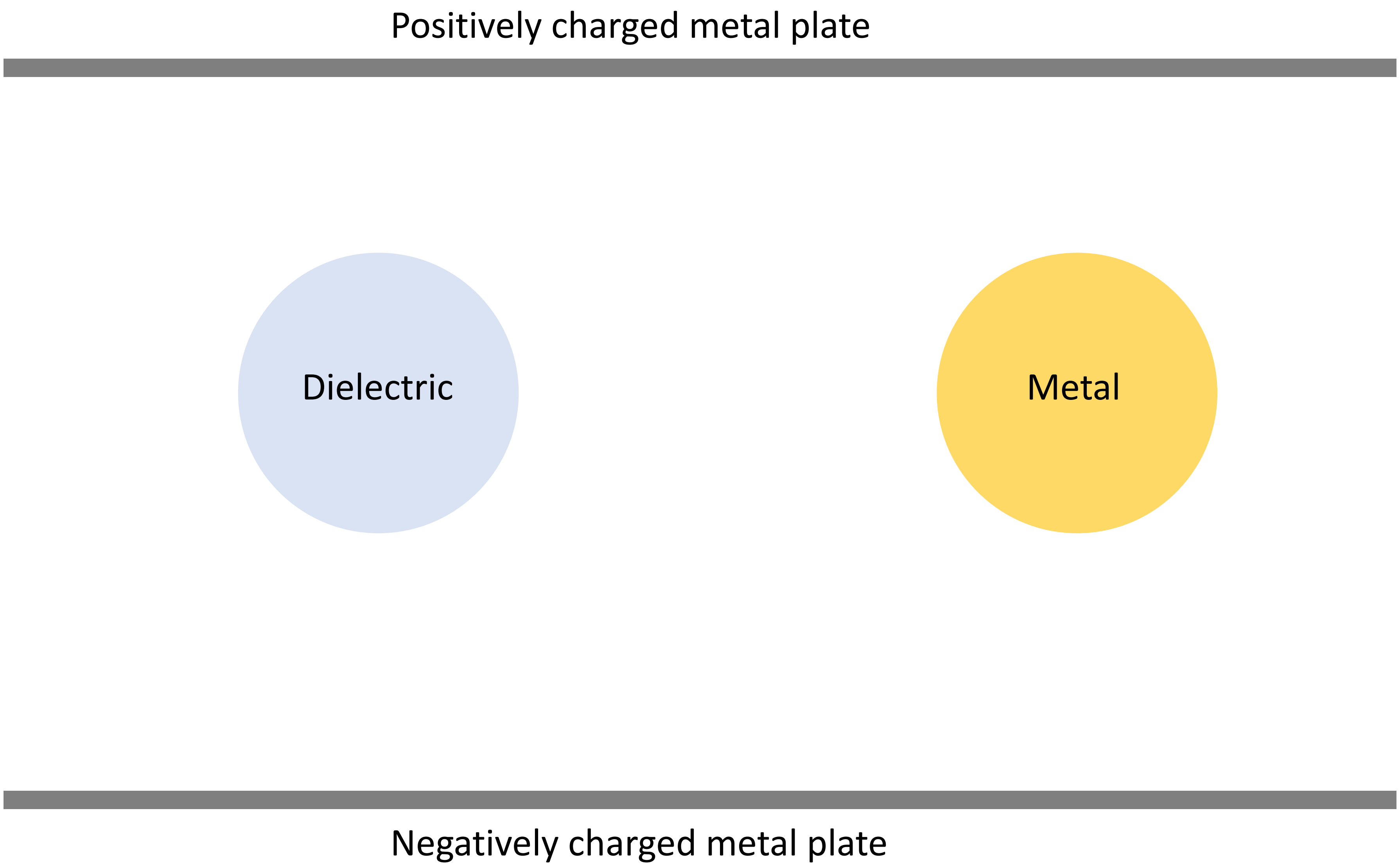
B. $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$

C. $\frac{1}{2}(|H\rangle\langle H| + |V\rangle\langle V|)$

D. $\frac{1}{2}(|H\rangle\langle H| + |V\rangle\langle V| + |H\rangle\langle V| + |V\rangle\langle H|)$

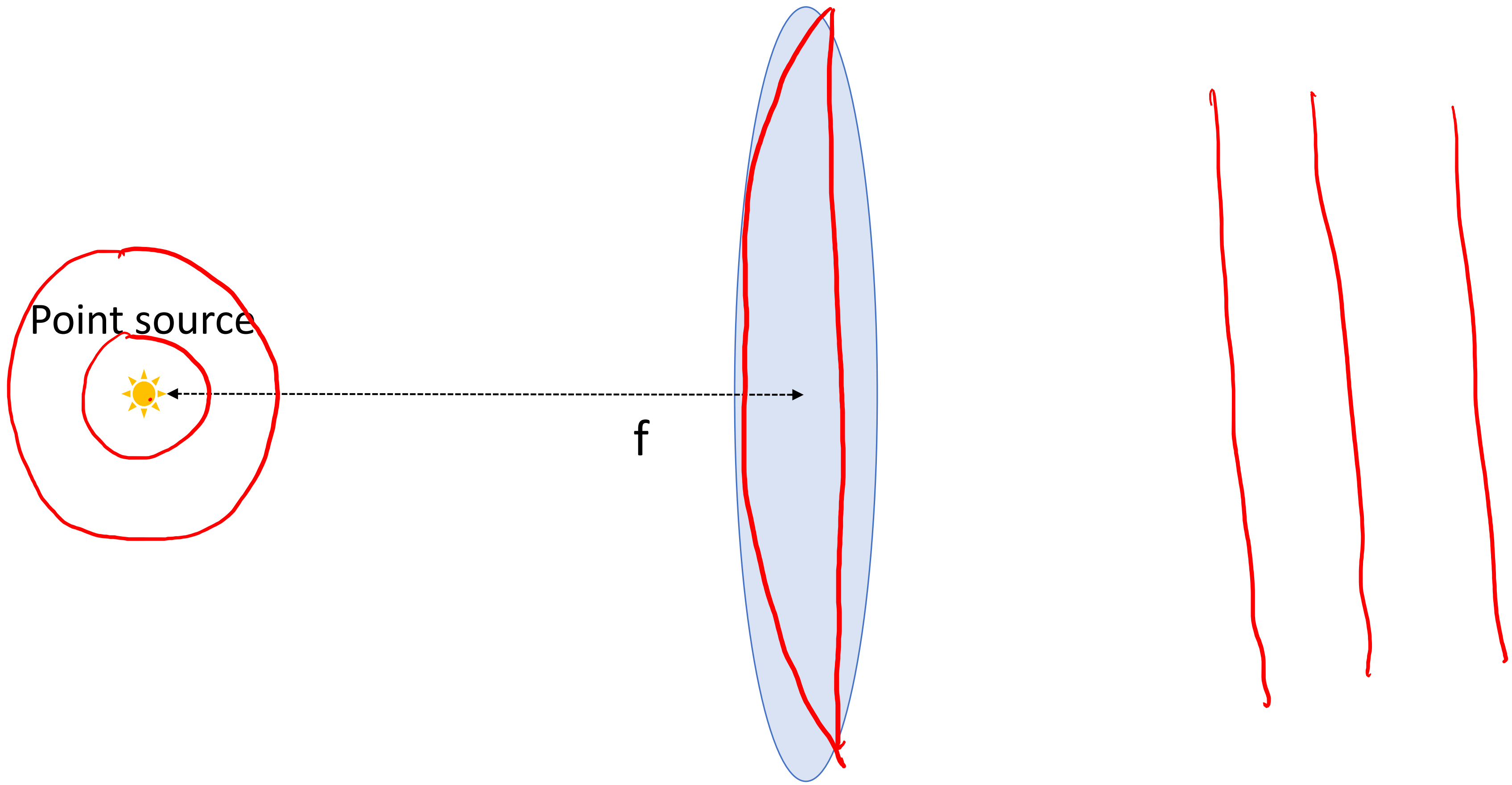
Exercise 1

Draw the field lines



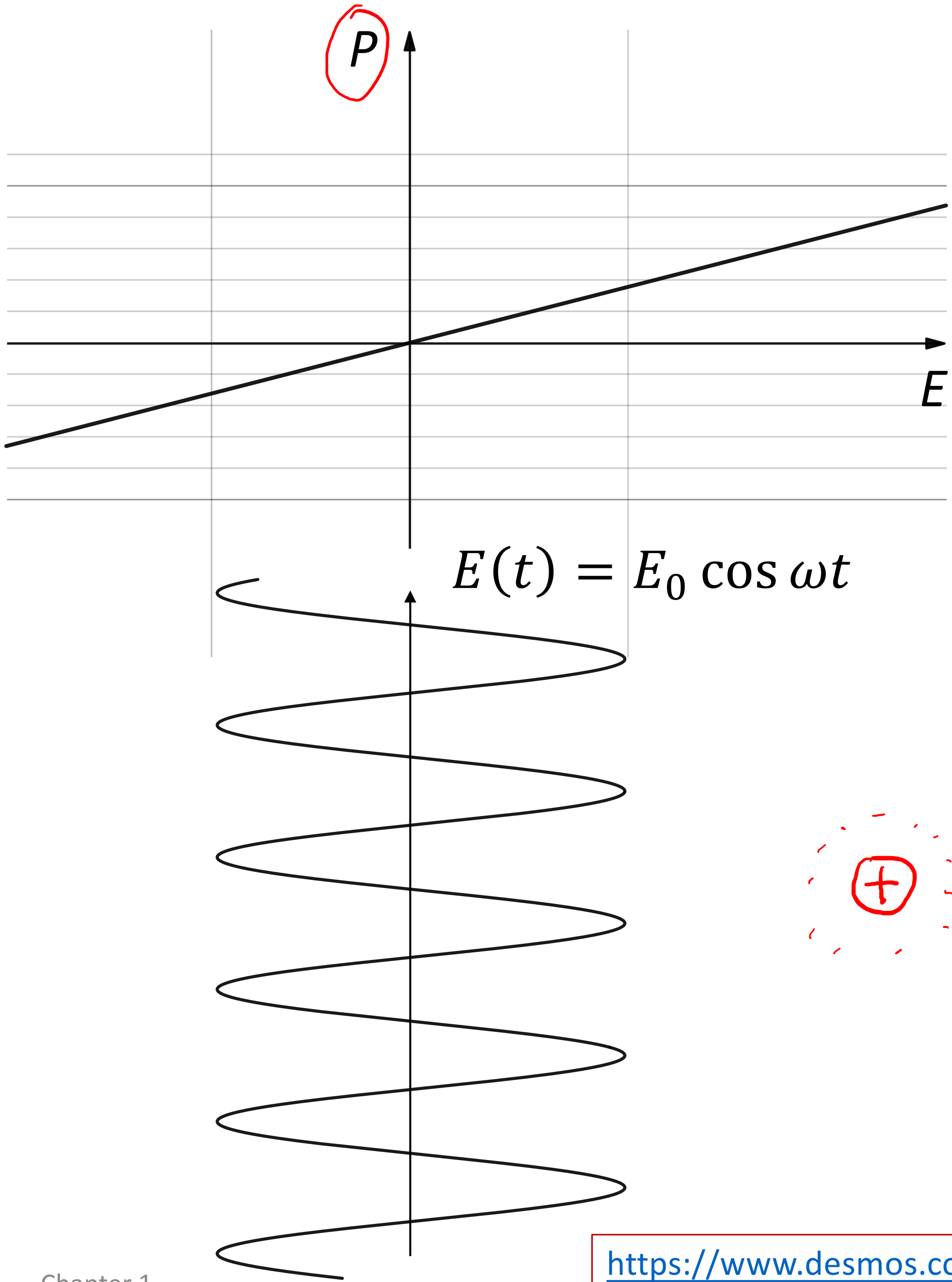
Exercise 2

Draw the wavefronts traveling to the right



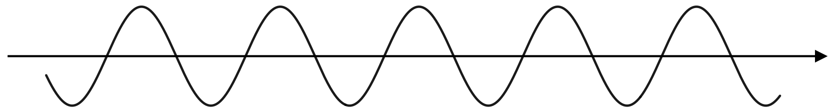
Week 01

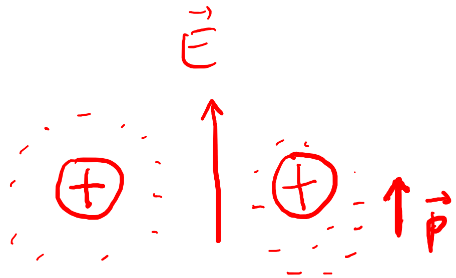
Introduction



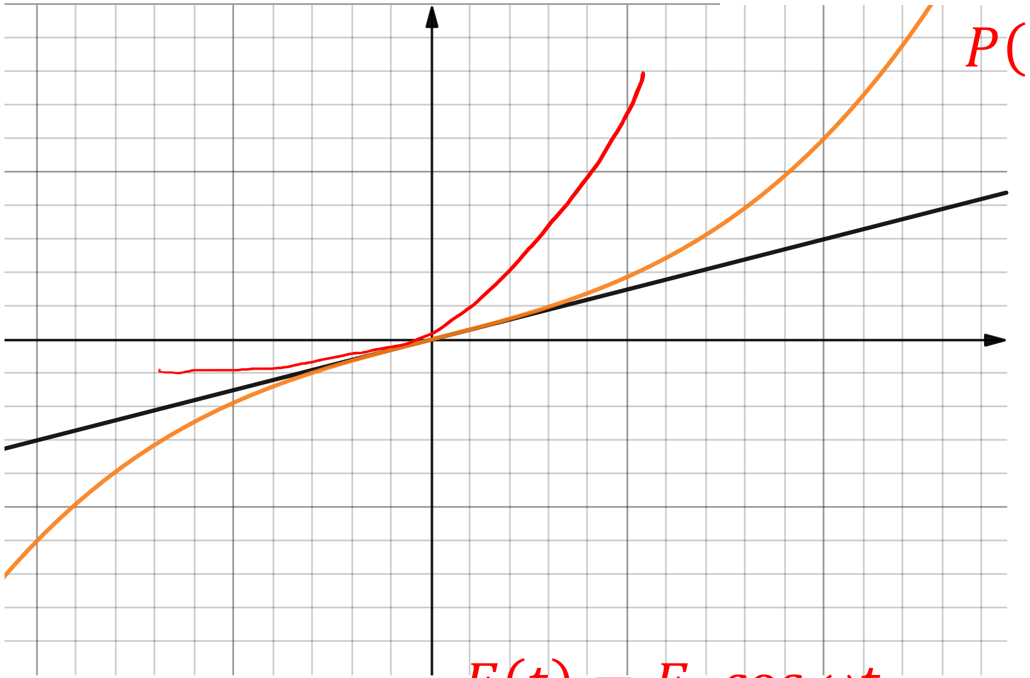
$$E(t) = E_0 \cos \omega t$$

$$P(t) = \varepsilon_0 \chi E_0 \cos \omega t$$

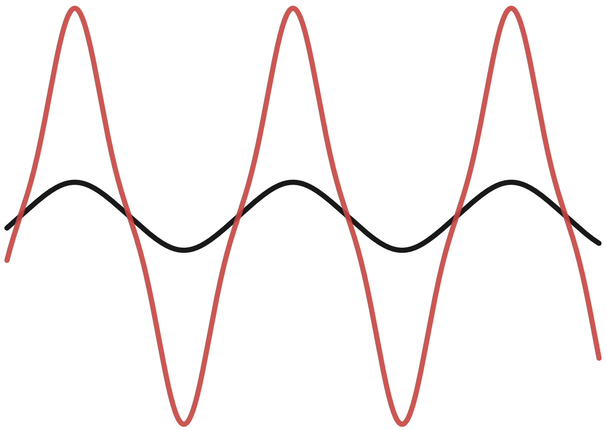




https://www.desmos.com/calculator/nsusz2dn4e λ cos ωt + μ cos...

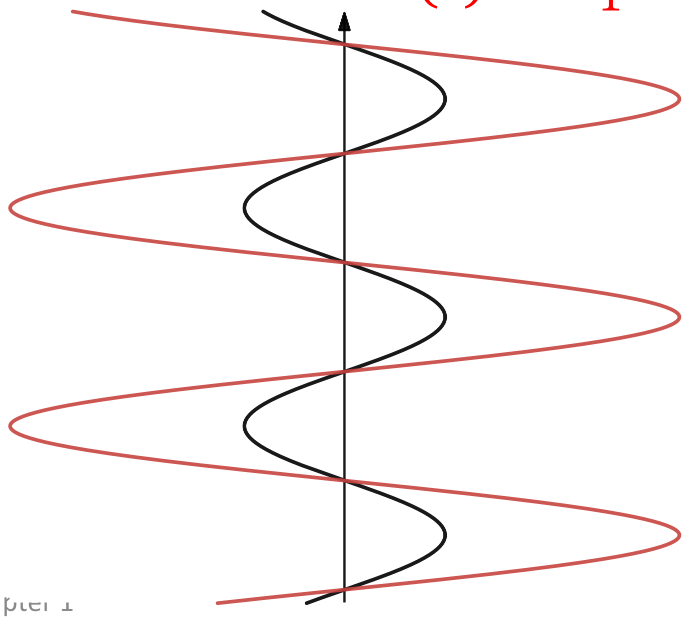


$$P(t) = \varepsilon_0 \chi(E_1 \cos \omega t + 0.01 \cdot E_1^3 \cos^3 \omega t)$$

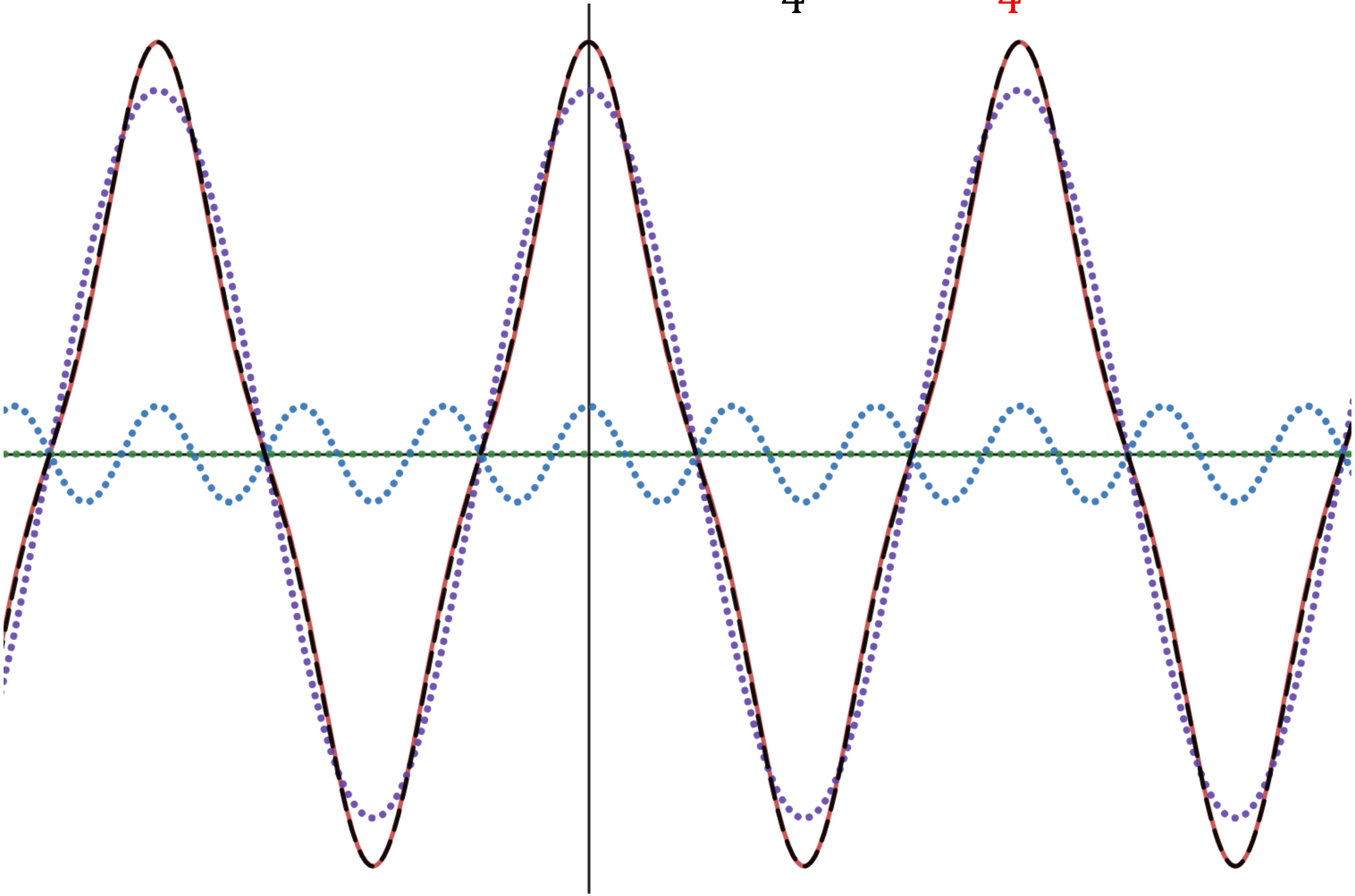


$$E(t) = E_1 \cos \omega t$$

$$\cos^3 \omega t = \frac{3}{4} \cos \omega t + \frac{1}{4} \cos 3\omega t$$



$$\cos^3 \omega t = \frac{3}{4}\cos \omega t + \frac{1}{4}\cos 3\omega t$$



https://www.desmos.com/calculator/1chtc1vjkz

A zoo of (classical) nonlinear processes…

Frequency-mixing processes[edit]

•Second-harmonic generation (SHG), or *frequency doubling*, generation of light with a doubled frequency (half the wavelength), two photons are destroyed, creating a single photon at two times the frequency.
•Third-harmonic generation (THG), generation of light with a tripled frequency (one-third the wavelength), three photons are destroyed, creating a single photon at three times the frequency.
•High-harmonic generation (HHG), generation of light with frequencies much greater than the original (typically 100 to 1000 times greater).
•Sum-frequency generation (SFG), generation of light with a frequency that is the sum of two other frequencies (SHG is a special case of this).
•Difference-frequency generation (DFG), generation of light with a frequency that is the difference between two other frequencies.
•Optical parametric amplification (OPA), amplification of a signal input in the presence of a higher-frequency pump wave, at the same time generating an *idler* wave (can be considered as DFG).
•Optical parametric oscillation (OPO), generation of a signal and idler wave using a parametric amplifier in a resonator (with no signal input).
•Optical parametric generation (OPG), like parametric oscillation but without a resonator, using a very high gain instead.
•Half-harmonic generation, the special case of OPO or OPG when the signal and idler degenerate in one single frequency,
•Spontaneous parametric down-conversion (SPDC), the amplification of the vacuum fluctuations in the low-gain regime.
•Optical rectification (OR), generation of quasi-static electric fields.
•Nonlinear light-matter interaction with free electrons and plasmas.[8][9][10][11]

Other nonlinear processes[edit]

•Optical Kerr effect, intensity-dependent refractive index (a effect).
   •Self-focusing, an effect due to the optical Kerr effect (and possibly higher-order nonlinearities) caused by the spatial variation in the intensity creating a spatial variation in the refractive index.
   •Kerr-lens modelocking (KLM), the use of self-focusing as a mechanism to mode-lock laser.
   •Self-phase modulation (SPM), an effect due to the optical Kerr effect (and possibly higher-order nonlinearities) caused by the temporal variation in the intensity creating a temporal variation in the refractive index.
   •Optical solitons, an equilibrium solution for either an optical pulse (temporal soliton) or spatial mode (spatial soliton) that does not change during propagation due to a balance between dispersion and the Kerr effect (e.g. self-phase modulation for temporal and self-focusing for spatial solitons).
   •Self-diffraction, splitting of beams in a multi-wave mixing process with potential energy transfer.[12]

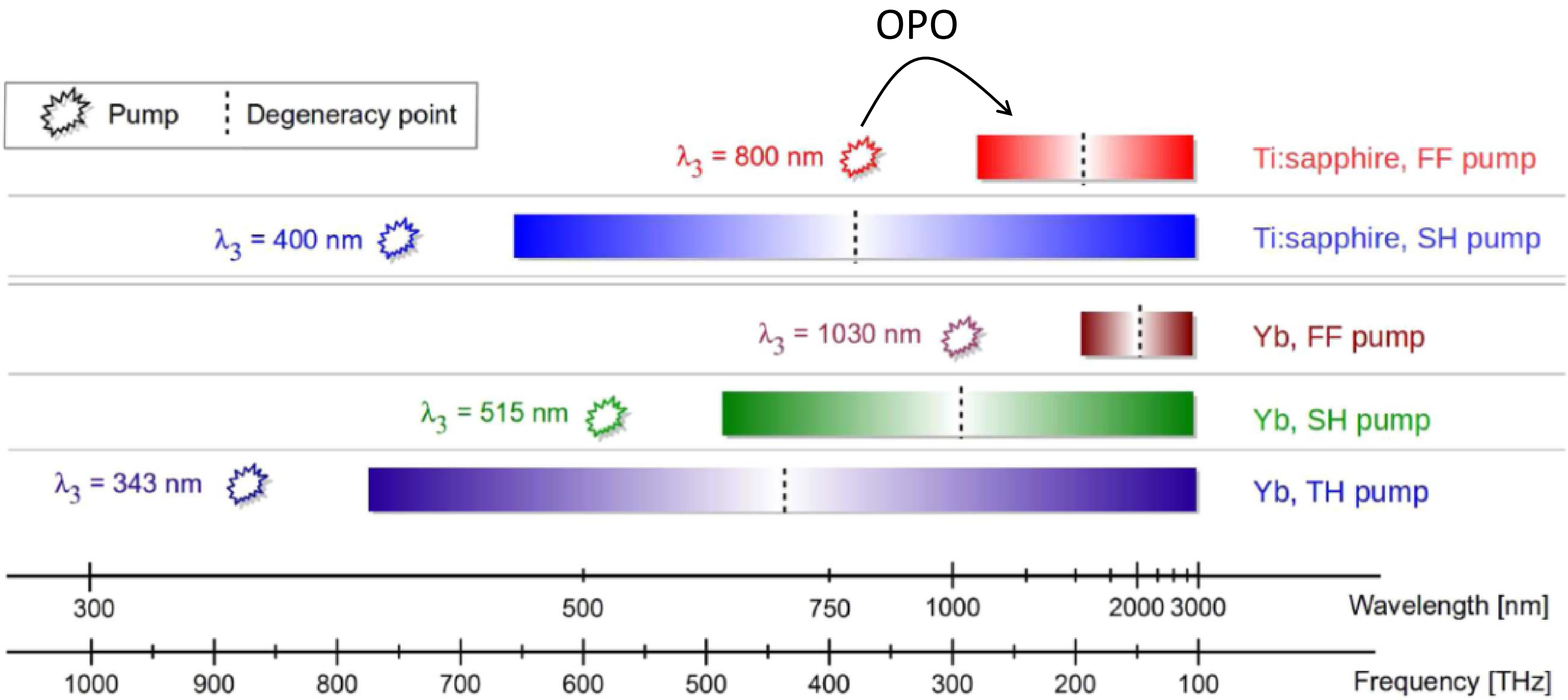
•Cross-phase modulation (XPM), where one wavelength of light can affect the phase of another wavelength of light through the optical Kerr effect.
•Four-wave mixing (FWM), can also arise from other nonlinearities.
•Cross-polarized wave generation (XPW), a effect in which a wave with polarization vector perpendicular to the input one is generated.
•Modulational instability.[13]
•Raman amplification[14]
•Optical phase conjugation.
•Stimulated Brillouin scattering, interaction of photons with acoustic phonons
•Multi-photon absorption, simultaneous absorption of two or more photons, transferring the energy to a single electron.
•Multiple photoionisation, near-simultaneous removal of many bound electrons by one photon.
•Chaos in optical systems.

Related processes[edit]

In these processes, the medium has a linear response to the light, but the properties of the medium are affected by other causes:
•Pockels effect, the refractive index is affected by a static electric field; used in electro-optic modulators.
•Acousto-optics, the refractive index is affected by acoustic waves (ultrasound); used in acousto-optic modulators.
•Raman scattering, interaction of photons with optical phonons.

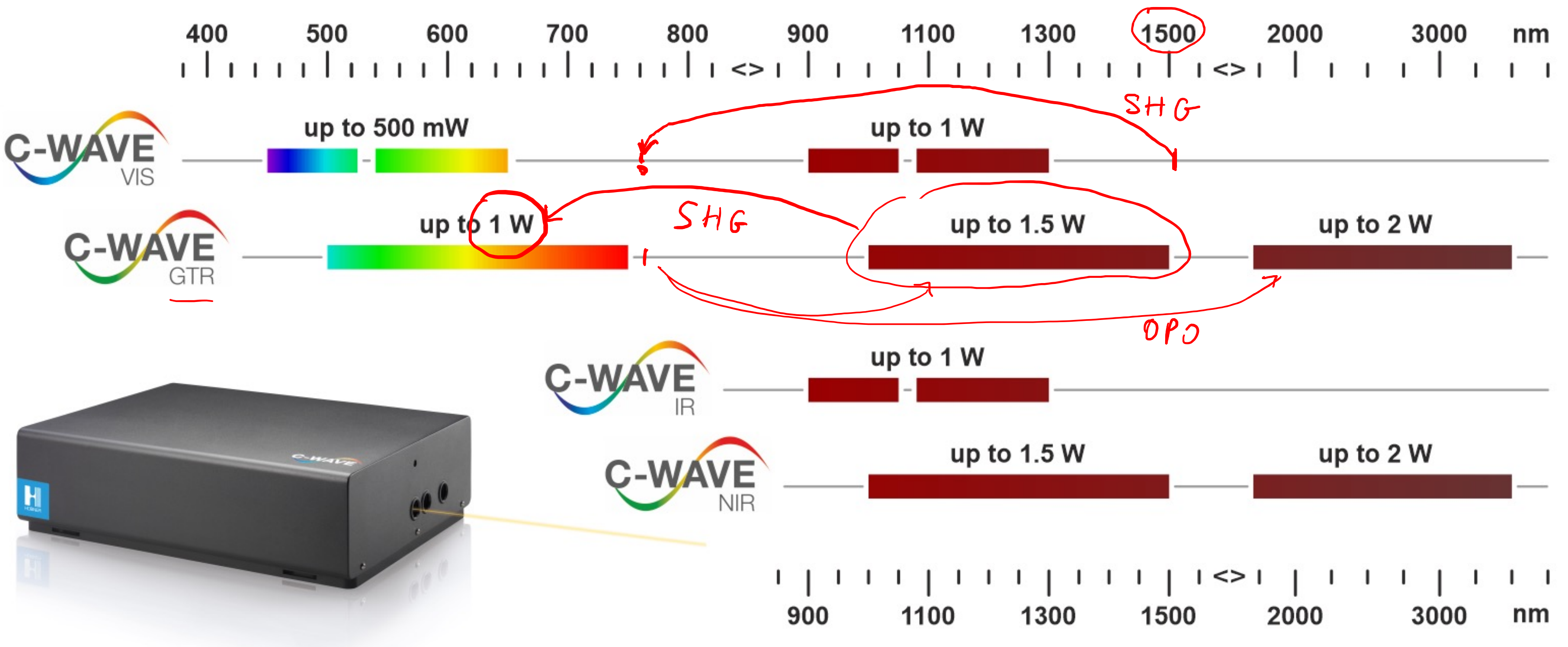
Light sources using nonlinear optics

- Extend the reach of coherent laser sources all over the electromagnetic spectrum

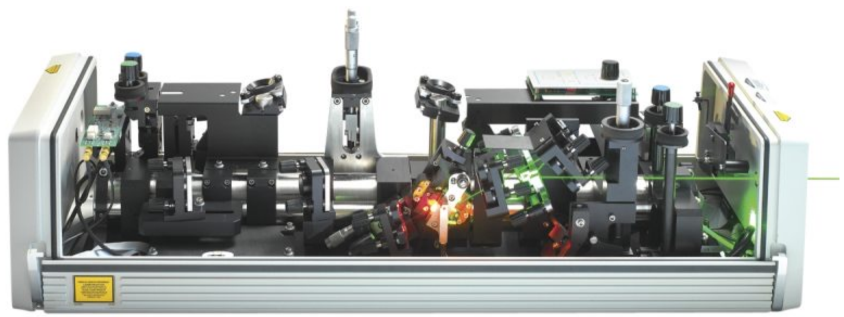


© Giulio Cerullo

In our labs…



https://hubner-photonics.com/products/lasers/tunable-lasers/c-wave/



https://www.spectra-physics.com/f/tsunami-ultrafast-oscillator

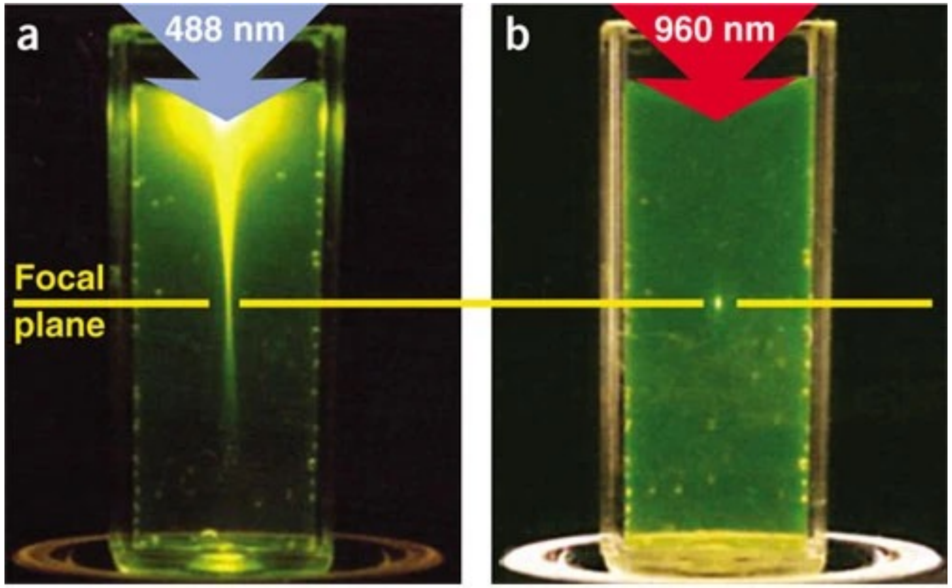
https://www.ape-berlin.de/en/opo-optical-parametric-oscillator/opo-x/

Spectroscopy with nonlinear optics

- Probe electronic and vibrational coherence in molecular and solid-state systems (e.g. “2D spectroscopy”, Coherent anti-Stokes Raman scattering...)

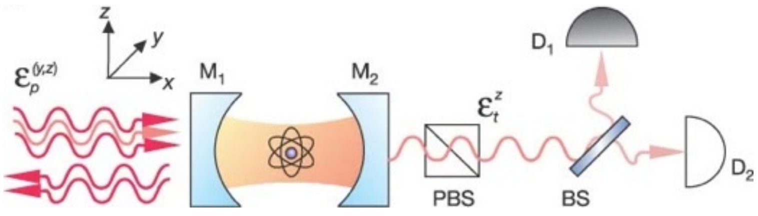
- Selectively probe interfaces (such as membranes, catalytic surfaces...)

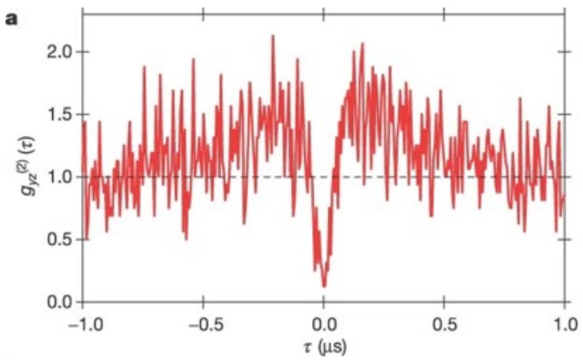
- Achieve super-resolution in microscopy (multiphoton spectroscopy, stimulated emission depletion, etc.)



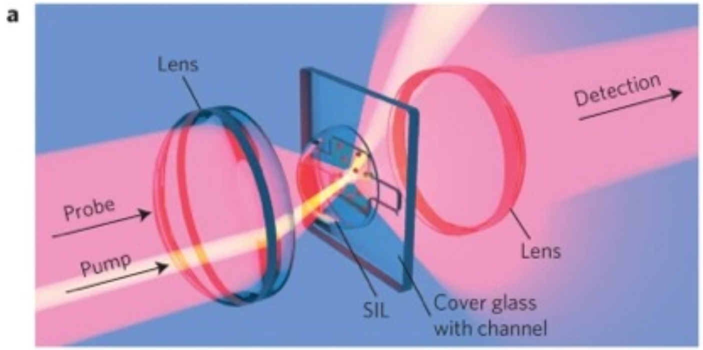
Single atom/molecule spectroscopy

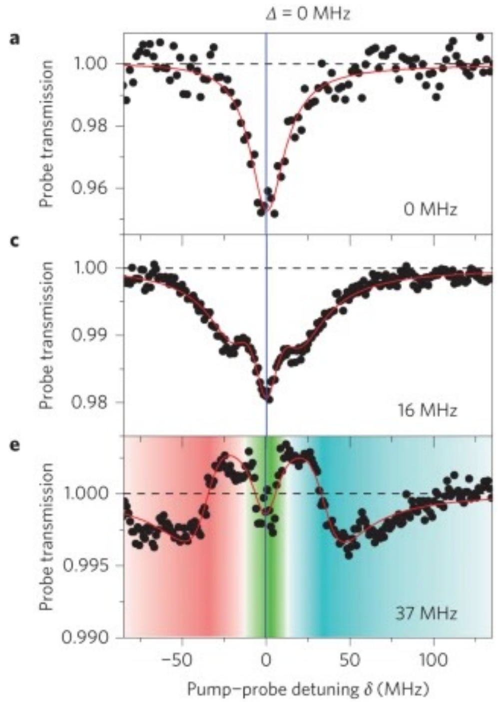
Photon blockade in an optical cavity with one trapped atom (2005)

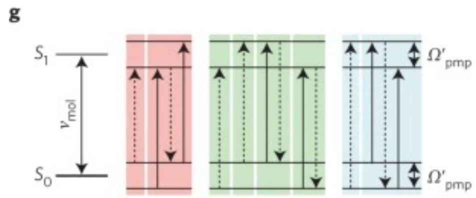




Few-photon coherent nonlinear optics with a single molecule (2016)







“Photon blockade in an optical cavity with one trapped atom” ***Nature*** 2005 https://www.nature.com/articles/nature03804

“Quantum nonlinear optics — photon by photon” (review) ***Nature Photonics*** 2014 https://www.nature.com/articles/nphoton.2014.192

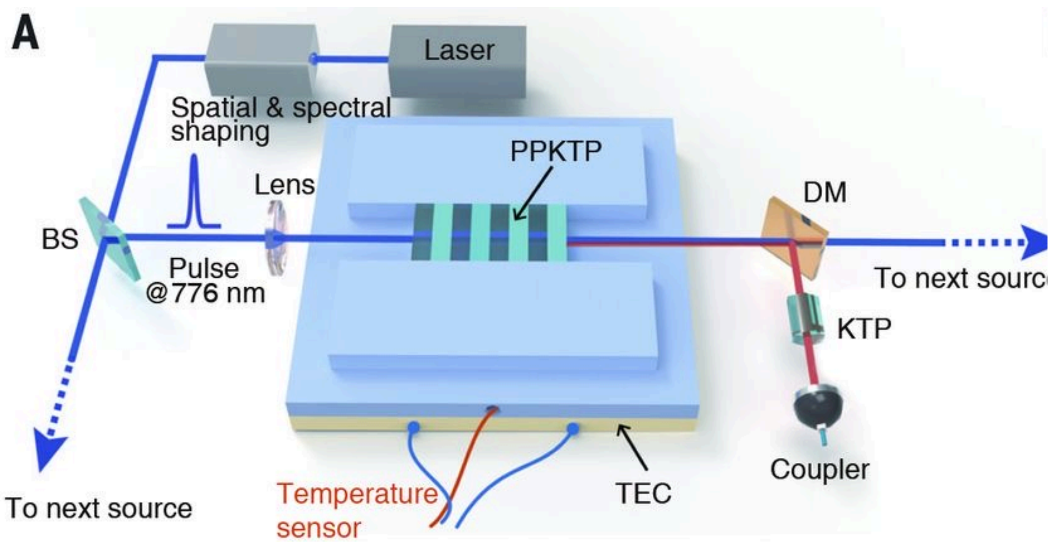
“Few-photon coherent nonlinear optics with a single molecule” ***Nature Photonics*** 2016 https://www.nature.com/articles/nphoton.2016.63

Vahid Sandoghdar group

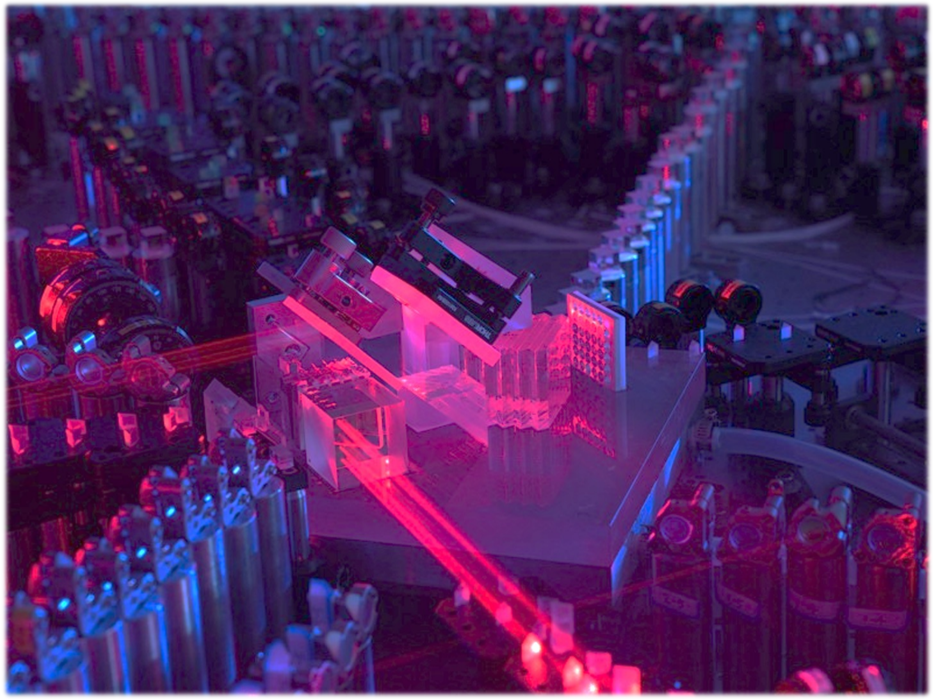
“Single-molecule vacuum Rabi splitting: four-wave mixing and optical switching at the single-photon level” https://arxiv.org/abs/2105.02560

Quantum applications of nonlinear optics

- **Generate states of light exhibiting non-classical properties, such as squeezing, entanglement, sub-Poissonian statistics, etc.** Such states are used for:
  - quantum key distribution and quantum communication,
  - quantum networks
  - quantum-enhanced sensing/imaging/metrology,
  - photonic quantum computing, quantum simulation

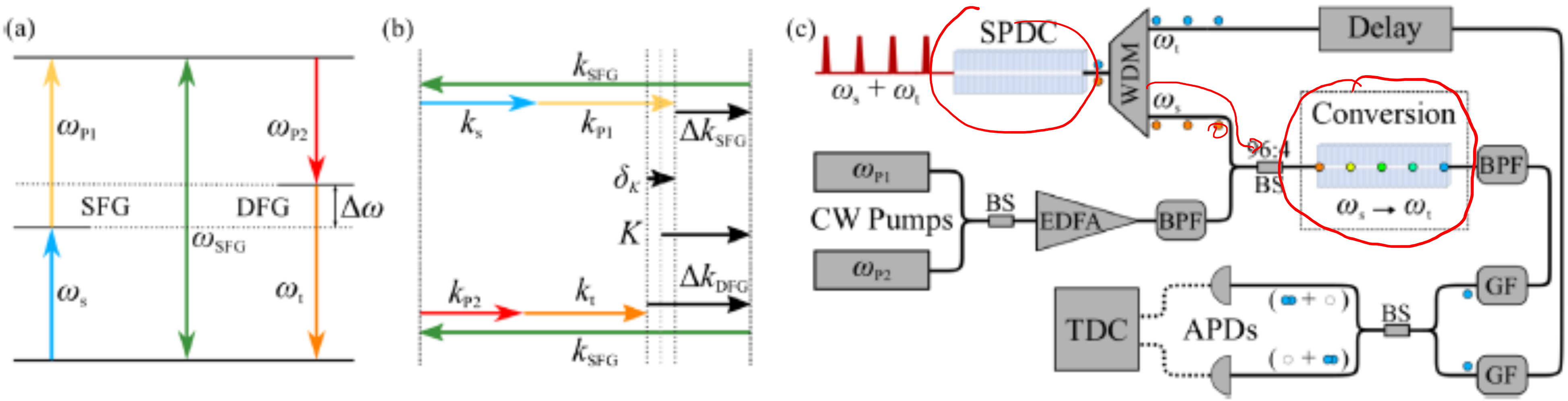


Quantum computational advantage using photons
Jian-Wei Pan's group; *Science* (03 Dec 2020)



Quantum applications of nonlinear optics

- **Quantum coherent frequency conversion**: map the quantum state of light from one frequency to another, preserving its wavefunction.



"Single Photon Frequency Conversion for Frequency Multiplexed Quantum Networks in the Telecom Band" ***Phys. Rev. Lett.*** (2021)
https://doi.org/10.1103/PhysRevLett.127.023602